A mixed integer optimization strategy for oil and gas production planning

A MIXED INTEGER OPTIMIZATION STRATEGY FOR OIL AND GAS PRODUCTION PLANNING

Dimitrios I. Gerogiorgis* and Efstratios N. Pistikopoulos
Centre for Process Systems Engineering, Imperial College London, SW7 2AZ, U.K.

Abstract

Oil and gas production is the cornerstone of the modern petrochemical industry, and its upstream as well as downstream processing provides many challenges to the process modeling, optimization and control areas. Mixed-integer optimization is a research field with a strong implementation record, having already been used to solve a wide spectrum of crude oil production, transport, distribution, planning and scheduling problems. Production optimization challenges are however perplexed by multiphase flow of oil, gas and water in the sub-surface circuits: the respective elements (reservoirs, wells) induce complexity in oil and gas transport which can only be handled suboptimally by use of linearized approximations of true pressure-flowrate curves. This paper addresses the problem of oil production maximization from a particular oilfield with several oil wells, all connected to one production platform and operating assisted by gas injection (secondary extraction). The proposed approach explicitly takes into account multiphase flow (based on a previously presented model) and relies on an MINLP model formulation toward calculating: (a) the operation (or shutting-in) of each well, (b) the volumetric flows of gas injection required in order to operate open production wells in gas-lift mode. An improved oil production optimum has been obtained for a case study considering a set of 6 gas-lift wells. This MINLP model can also be used for multiperiod optimization under additional cost and price constraints.

Keywords

Oil, gas, production planning, gas-lift wells, Mixed-Integer Nonlinear Programming (MINLP).

Introduction

Petroleum is by far the most valued natural resource: either as energy carrier or as process feedstock, its availability is always of paramount importance to the chemical industry. Field exploration, oil production, transport and processing operations provide an expanding portfolio of challenges. Many modeling and optimization problems have thus been tackled in both upstream and downstream processing areas: gas-lift well operation (Alarcón et al., 2002), oilfield infrastructure planning (Van den Heever and Grossmann, 1999; Van den Heever et al., 2000; Lin and Floudas, 2003), gas field development (Goel et al., 2006), oil scheduling (Shah, 1996), oil distribution (Más and Pinto, 2003), and planning and scheduling of the many process operations in refinery complexes (Moro et al., 1998; Pinto et al., 2000). Multidisciplinary insights into geological complexity and multiphase flow in porous strata can also provide benefits.

The time-dependent performance of each production well is monitored by the characteristic pressure-flowrate curve: its effect has been explicitly considered in determining the optimal well oil rate allocation (Kosmidis et al., 2004) as well as optimal well scheduling (Kosmidis et al., 2005). Mathematical modeling of sub-surface elements increases accuracy and reliability (Barragán-Hernández et al., 2005). Further model integration can thus employ multiphase flow simulations for enhancing operations (e.g. selectively assist gas production from gas-rich or even oil-depleted reserves). Upper-level optimization gains from low-level multiphase simulation of oil, gas and water flow: indeed, a dynamic reservoir flow simulator (ECLIPSE®) has been combined with an equation-oriented process optimizer (gPROMS®), thus yielding more accurate valve settings and production resource allocations (Gerogiorgis and Pistikopoulos, 2006).

* To whom all correspondence should be addressed at D.Gerogiorgis@imperial.ac.uk

Petroleum production: the operation of gas-lift wells

Petroleum production platforms house many elements, all essential to primary (natural) and secondary (gas-lift) oil production: all are illustrated in Figure 1 (Biggs, 1975).

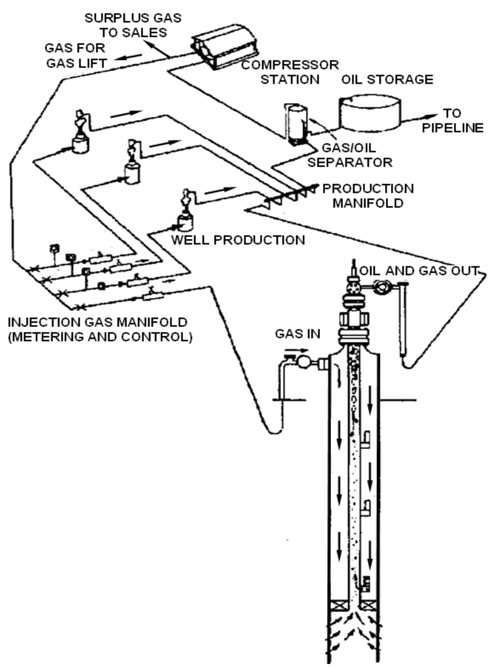

Figure 1. Operation of gas-lift oil production wells.

The effect of gas injection rate on oil production rate for different cases of gas-lift wells is illustrated in Figure 2: the most responsive wells react immediately to gas lift, yielding oil flow even for very low gas injection (curve B); conversely, mature wells require a minimum gas injection flowrate before they even begin to produce oil (curve C). In both cases, oil production flow can only be increased up to a maximum, beyond which gas lift is no longer viable.

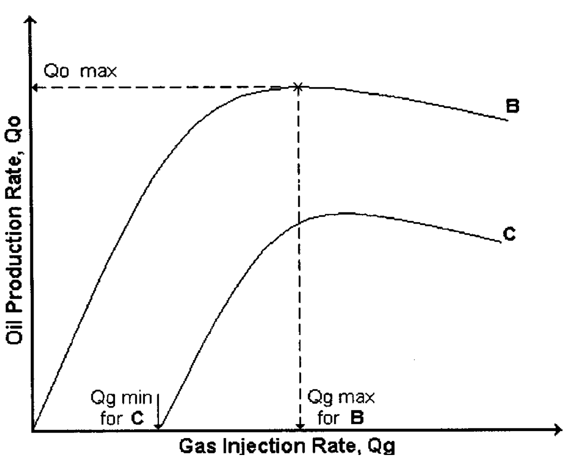

Figure 2. Two typical gas-lift performance curves.

Mathematical model of multiphase flow in wells

Explicit reservoir flow simulation can be employed for enhancing the accuracy of production well performance models, using a dynamic reservoir simulator (ECLIPSE®). Resulting pressure-flowrate data can be readily used by an equation-oriented process optimizer (GAMS®, gPROMS®) in order to determine optimal reservoir operation setpoints. A relevant methodology for optimal production planning is found in an upcoming publication (Gerogiorgis et al., 2008).

Table 1. Multiphase flow model for reservoir simulation.

Oil phase distribution

$$\nabla[\frac{k\,k_{ro}}{\mu_o B_o}\nabla(P_o+\rho g h)]+q_o=\frac{\partial}{\partial t}(\varphi\frac{S_o}{B_o}) \tag{1}$$

Water phase distribution

$$\nabla[\frac{k\,k_{rw}}{\mu_w B_w}\nabla(P_w+\rho g h)]+q_w=\frac{\partial}{\partial t}(\varphi\frac{S_w}{B_w}) \tag{2}$$

Gas phase distribution

$$\nabla[\frac{k\,k_{rg}}{\mu_g B_g}\nabla(P_g+\rho g h)]+\nabla[R_s\frac{k\,k_{ro}}{\mu_o B_o}\nabla(P_o+\rho g h)]+ \\ +q_g=\frac{\partial}{\partial t}(\phi\frac{S_g}{B_g}+R_s\frac{\phi\,S_o}{B_o}) \tag{3}$$

Total pressure gradient calculation

$$\frac{dP}{dx}=-g\rho_m(x)\sin(\theta)-\frac{\tau_w(x)S}{A} \tag{4}$$

Capillary pressure (oil/gas interface)

$$P_{cog}(S_o,S_g)=P_o-P_g \tag{5}$$

Capillary pressure (oil/water interface)

$$P_{cow}(S_o,S_w)=P_o-P_w \tag{6}$$

Multiphase mixture saturation closure

$$S_o+S_w+S_g=1 \tag{7}$$

Multiphase mixture density

$$\rho_m(x)=\rho_l(x)E_l(x)+\rho_g(x)E_g(x) \tag{8}$$

Multiphase mixture viscosity

$$\mu_m(x)=\mu_l(x)E_l(x)+\mu_g(x)E_g(x) \tag{9}$$

Multiphase mixture superficial velocity

$$U_m(x)=\frac{\rho_l(x)}{\rho_m(x)}U_{sl}(x)+\frac{\rho_g(x)}{\rho_m(x)}U_{sg}(x) \tag{10}$$

Multiphase mixture holdup closure

$$E_g(x)+E_l(x)=1 \tag{11}$$

Drift flux model (gas holdup)

$$E_g=f_d(\rho_l,\rho_g,\rho_m,\mu_l,\mu_g,\mu_m,U_{sl},U_{sg},U_m) \tag{12}$$

Choke model (for well & valve i)

$$q_{l,i}=f_c(d_i,P_i(x_{ch}^-),P_i(x_{ch}^+),c_i,q_{g,i},q_{w,i}) \tag{13}$$

Well phase flowrate as a function of saturations

$$q_{p,i}=\sum_p S_{p,i}q_{l/g,i}\,,\forall p\in\{o,w,g\},\forall i\in I\{gas\ lift\} \tag{14}$$

Optimization model for efficient gas-lift well operation

The proposed model includes the following elements, as explained in our methodology (Gerogiorgis et al., 2008): (i) the *objective function* (maximization of oil production), (ii) the *multiphase flow model*, (iii) the *mass, momentum and energy balances* in the set of gas-lift production wells, (iv) the *logic constraints* for operating or shutting-in wells.

The simplifying assumptions introduced here follow: (i) there are only gas-lift production wells in the oil field, (ii) all wells are independently connected to the platform, (iii) only a single liquid-gas phase separator is available, (iv) gas injection is local in each gas-lift well (Figure 1), and does not significantly affect (either in space or in time) the underlying multiphase flow and the reservoir dynamics.

The objective function and the most crucial constraints for the case study considered here are presented in Table 2.

Table 2. Optimization model for gas-lift oil production.

Oil production maximization (objective function)

$$\max_{y_i, q_{g,i}^{inj}} \; q_o = \sum_i q_{o,i} \,, \qquad \forall i \in I\{gas\ lift\} \tag{15}$$

subject to the constraints:

Total gas flowrate (flowing + injection)

$$q_{g,i} = q_{g,i}^{f} + q_{g,i}^{inj}, \qquad \forall i \in I\{gas\ lift\} \tag{16}$$

Total liquid (oil and water) flowrate

$$q_{l,i} = \sum_p q_{p,i}, \forall p \in \{o, w\}, \qquad \forall i \in I\{gas\ lift\} \tag{17}$$

Well oil flowrate bounds

$$y_i \, q_{o,i}^{L} \le q_{o,i} \le y_i \, q_{o,i}^{U}, \qquad \forall i \in I\{gas\ lift\} \tag{18}$$

Well oil flowrate vs. gas injection (Alarcón et al., 2002)

$$q_{o,i} = \sum_{n=0}^{2} a_n \left(q_{g,i}^{inj}\right)^n + a_3 \ln(q_{g,i}^{inj} + 1), \forall i \in I\{gas\ lift\} \tag{19}$$

Well gas injection bounds

$$y_i \, q_{g,i}^{inj\ L} \le q_{g,i}^{inj} \le y_i \, q_{g,i}^{inj\ U}, \qquad \forall i \in I\{gas\ lift\} \tag{20}$$

Total gas injection capacity bound

$$\sum_i q_{g,i}^{inj} \le C_c \,, \qquad \forall i \in I\{gas\ lift\} \tag{21}$$

Total enthalpy (per well flowline)

$$H_i^m = \sum_p S_{p,i} H_{p,i} \,, \forall p \in \{o, w, g\}, \ \forall i \in I\{gas\ lift\} \tag{22}$$

Well flowline enthalpy bounds

$$0 \le H_i^m \le y_i \, H^U, \qquad \forall i \in I\{gas\ lift\} \tag{23}$$

Oil phase separator capacity bounds

$$\sum_i q_{o,i} \le C_o \,, \qquad \forall i \in I\{gas\ lift\} \tag{24}$$

Water phase separator capacity bounds

$$\sum_i q_{w,i} \le C_w \,, \qquad \forall i \in I\{gas\ lift\} \tag{25}$$

Gas phase separator capacity bounds

$$\sum_i q_{g,i} \le C_g \,, \qquad \forall i \in I\{gas\ lift\} \tag{26}$$

Case study and results

The case study selected for evaluating the proposed approach is adapted from a petroleum engineering paper (Alarcón et al., 2002) which uses the classic approximation of *pressure-flowrate curves* (as in Kosmidis et al., 2004), but considers: (a) continuous nonlinear ($q_{o,i}$ vs. $q_{g,i}^{inj}$) curves, (b) a systematic NLP optimization method (SQP algorithm), rather than use of *equal slope* heuristics (Kanu et al., 1981). Therein, all systematically determined oil production optima are lower than those calculated by the *equal slope* heuristic, yet they are proved more consistent to production well data.

The present paper considers the same example problems (5 type-B with/without 1 type-C gas-lift oil wells – Fig. 2). The novelty here is that we compute state variable profiles (pressure, oil-gas-water saturation, flows) via Eqs. (1)-(14), using a reservoir multiphase flow simulator (ECLIPSE®), via realistic rock permeability data from a built-in database; here, we model the reservoir but not the wells themselves. Thus, the use of phase distribution correlations is avoided (employing the underlying *black-oil* thermodynamic model), and the flows and saturations computed by Eq. (1)-(3)+(14) can be utilized in the MINLP formulation of Eqs. (15)-(29). Asynchronous interfacing is possible when assuming that all gas-lift production wells are located at reasonable distances.

Oil production and well operation results (Table 3) refer to 2 different cases, under gas injection capacity bounds (C_c).

Table 3. Oil well operation and optimal production results.

Well (i)	Operation† (y_i)	$q_{g,i}^{inj}$ (std m^3.d^{-1})	$q_{o,i}$ (m^3.d^{-1})	$q_{g,i}^{inj}$ (std m^3.d^{-1})	$q_{o,i}$ (m^3.d^{-1})
C_c=84950.6 m^3.d^{-1}		(Alarcón et al., 2002)		†This Work	
1	$y_1 = 1$	7455.8	51.7	7458.6	51.9
2	$y_2 = 1$	15786.6	114.7	15790.2	114.8
3	$y_3 = 1$	25037.8	172.1	25069.3	172.3
4	$y_4 = 1$	16588.0	92.4	16603.1	93.1
5	$y_5 = 1$†	20082.3	107.7	20029.3	108.1
Total	**Σ**	**84950.6**	**538.6**	**84950.6**	**540.2**
Well (i)	Operation‡ (y_i)	$q_{g,i}^{inj}$ (std m^3.d^{-1})	$q_{o,i}$ (m^3.d^{-1})	$q_{g,i}^{inj}$ (std m^3.d^{-1})	$q_{o,i}$ (m^3.d^{-1})
C_c=130257.5 m^3.d^{-1}		(Alarcón et al., 2002)		‡This Work	
1	$y_1 = 1$	9844.4	54.4	15543.0	60.6
2	$y_2 = 1$	17457.6	116.7	21287.9	121.6
3	$y_3 = 1$	27814.5	175.4	34254.8	183.1
4	$y_4 = 1$	18910.0	95.1	24606.3	101.8
5	$y_5 = 1$	23859.5	112.1	34565.5	123.3
6	$y_6 = 0$‡	32371.8	14.7	0	0
Total	**Σ**	**130257.5**	**568.5**	**130257.5**	**590.4**

These optimization results indicate that shutting-in the most expensive (in terms of gas injection flowrate) oil wells can indeed yield an improved total oil production maximum. Even when shutting-in is found impossible (first example), a marginal improvement can be achieved in oil production.

Conclusions

Integrated modeling and optimization of oil and gas production is a research trend aiming to bridge the gap between the *classic paradigm in petroleum engineering* (detailed study of rock permeability and multiphase flow) and *prevalent process systems engineering methodologies* (systematic multiperiod or dynamic optimization models). Oil reservoirs, wells and surface facilities are thus regarded as *a single yet multiscale process system*, whose operation can be optimized by discrete (e.g. open/closed well valves) and continuous (e.g. gas injection rates) decision variables. A literature problem solved shows that using flow results yields an improved oil production maximum, due to the enhanced accuracy achieved in flow and phase distribution. An augmented multiperiod MINLP optimization model can use an economic objective function (profit maximization) in order to determine optimal production planning under more (transport cost, oil vs. gas demand, price) constraints.

Nomenclature

Latin letters

A	surface area	m^2
C	facility capacity	W
E	holdup	dimensionless
g	gravitational acceleration	$m.s^{-2}$
h	height	m
k	absolute permeability	Darcy ($\sim 10^{-6}$ m^2)
k_r	relative permeability	dimensionless
P	pressure	Pa ($N.m^{-2}$)
q	flow	$(bbl.d^{-1})_o$ $(std.\ m^3.d^{-1})_g$
S	phase saturation	dimensionless
t	time	s
U	fluid velocity	$m.s^{-1}$
x	position	m
y	well valve setting (on/off)	binary variable

Greek letters

θ	inclination angle	rad
μ	viscosity	$kg.m^{-1}.s^{-1}$
ρ	density	$kg.m^{-3}$
φ	rock porosity	dimensionless

Subscripts

c	compressor	m	mixture
g	gas	o	oil
i	well index	r	relative
j	phase index	s	superficial
l	liquid	w	water

Acknowledgements

The authors acknowledge financial support as well as a postdoctoral fellowship from the European Union (FP6) under a Marie Curie Research Training Network (RTN): “Towards Knowledge-Based Processing Systems” (PRISM) (EU RTN Project: MRTN-CT-2004-512233, 2004-2008).

References

Alarcón, G.A., Torres, C.F., Gómez, L.E., “Global optimization of gas allocation to a group of wells in artificial lift using nonlinear constrained programming,” *Trans. ASME: J. Ener. Res. Tech.* **124**(12): 262-268 (2002).

Barragán-Hernández, V., Vázquez-Román, R., Rosales-Marines, L., García-Sánchez, F., “A strategy for simulation and optimization of gas and oil production,” *Comput. Chem. Eng.* **30**(2): 215-227 (2005).

Biggs, M.C., “Constrained minimization using recursive quadratic programming,” in: *Towards Global Optimization* (L. Dixon & G.P. Szergo, eds.), North Holland (1975).

GeoQuest, *ECLIPSE® 300 Technical Description (version 2000A)*, GeoQuest, Schlumberger SCR, Cambridge, UK (2000).

Gerogiorgis, D.I., Kosmidis, V.D., Pistikopoulos, E.N., “Mixed integer optimization in well scheduling,” (in press), in: *Encyclopedia of Optimization – 2nd Edition* (C.A. Floudas & P.M. Pardalos, eds.), Springer (2008).

Gerogiorgis, D.I., Pistikopoulos, E.N. et al., “Dynamic oil and gas production systems optimization via explicit reservoir and well multiphase flow simulation”, in: *Proceedings of ESCAPE-16/PSE2006* (W. Marquardt & C.C. Pantelides, eds.), pp. 179-184, Elsevier, Amsterdam, ND (2006).

Goel, V., Grossmann, I.E., El-Bakry, A.S., Mulkay, E.L., “A novel branch and bound algorithm for optimal development of gas fields with uncertainty in reserves,” *Comput. Chem. Eng.* **30**(6-7): 1076-1092 (2006).

Kanu, E.P., Mach, J., Brown, K.E., “Economic approach to oil production and gas allocation in continuous gas lift,” *J. Pet. Technol.* **33**(10): 1887-1892 (1981).

Kosmidis, V.D., Perkins, J.D., Pistikopoulos, E.N., “Optimization of well oil rate allocations in petroleum fields,” *Ind. Eng. Chem. Res.* **43**(14): 3513-3527 (2004).

Kosmidis, V.D., Perkins, J.D., Pistikopoulos, E.N., “A mixed integer optimization formulation for the well scheduling problem on petroleum fields,” *Comput. Chem. Eng.* **29**(7): 1523-1541 (2005).

Lin, X., Floudas, C.A., “A novel continuous-time modeling and optimization framework for well platform planning problems,” *Optim. & Eng.* **4**(1-2): 65-95 (2003).

Más, R., Pinto, J.M., “A mixed-integer optimization strategy for oil supply in distribution complexes,” *Optim. & Eng.* **4**(1-2): 23-64 (2003).

Moro, L.F.L., Zanin, A.C., Pinto J.M., “A planning model for refinery diesel production,” *Comput. Chem. Eng.* **22(S)**: S1039-S1042 (1998).

Pinto, J.M, Joly, M., Moro, L.F.L., “Planning and scheduling models for refinery operations,” *Comput. Chem. Eng.* **24**(9): 2259-2276 (2000).

Shah, N., “Mathematical programming techniques for crude oil scheduling,” *Comput. Chem. Eng.* **20(S)**: S1227-S1232 (1996).

Van den Heever, S.A. and Grossmann, I.E., “An iterative aggregation/disaggregation approach of a mixed-integer nonlinear oilfield infrastructure planning model”, *Ind. Eng. Chem. Res.* **39**(6): 1955-1971 (2000).

Van den Heever, S.A., Grossmann, I.E., Vasantharajan, S., Edwards, K., “Integrating complex economic objectives with the design and planning of offshore oilfield infrastructures,” *Comput. Chem. Eng.* **24**(2-7): 1049-1055 (2000).